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Michael Markov, CEO  
Markov Processes International  
Tel +1 908 608 1558  
[www.markovprocesses.com](http://www.markovprocesses.com)

## Madoff: A Tale of Two Funds

**A quantitative analysis of a Madoff hedge fund reveals a striking similarity with another well-known case of fraud**

### Background

Now that the news of Madoff's \$50B Ponzi scheme is on the front pages of news media around the world, the investment community is scratching their heads and trying to understand how this could have happened. With all the advances in academic research, investment technology and risk management tools it seems impossible to pull off an enterprise of such dimensions. The truth is that investors often prefer to ignore red flags and forgo analytical tools and techniques specifically designed to protect them from such a fraud. In 2006, at the request of a hedge fund manager, MPI performed a returns-based quantitative analysis of one of Madoff's funds and came to the conclusion that, most likely, the returns were not real. Moreover, in trying to explain the strategy we discovered that the fund's return pattern resembled that of a proven fraud case – the Bayou fund. This research article follows the footsteps of our 2006 study.

### Returns-Based Forensics

Since 1992<sup>1</sup> returns-based style analysis ("RBSA") tools have become firmly entrenched in the investment due diligence process on the traditional side of the industry. Institutional investors are presented with choices either of fully trusting portfolio managers or of performing tests to understand whether the manager's stories matched the results. Up until the early 90's this was a formidable and expensive exercise requiring the help of consultants or of investing heavily in holdings-based analytics technology and associated infrastructure to perform a laborious analysis of the portfolio's entire holdings history. With the advent of RBSA the process of reconciling performance with the stated management strategy became quick, inexpensive and very accurate.

This methodology gained widespread acceptance within the traditional side of the industry and provided

<sup>1</sup> In 1992 MPI was the first to develop returns-based style analysis (RBSA) tool following the ideas of Nobel Prize winner in Economics Prof. William Sharpe.

investors with inexpensive and effective means to understand inner workings of investment products by using only performance data. It is worth mentioning that such tools are used alongside holdings-based analytics as it is a known that holdings may not be able to tell the whole story. This is because portfolio managers frequently use "window dressing," use complex derivative strategies such as "portable alpha," which carry heavy risks, and, finally, may not provide correct and full position information. The idea behind the returns-based approach is relatively simple. On one hand there is a holding-based story describing the investment strategy and the instruments used. On the other hand there's a track record—a stream of monthly investment returns—that can often be closely mimicked by finding a combination of passive factors or indices that best explain the return movements. If the dynamics of factor/index exposures agree with the information derived from holdings, it reinforces the confidence in the strategy. However, if there's a notable discrepancy, it enables an investor to question the manager's story.

Until recently, the applicability of returns-based analysis to alternative investment products was relatively limited. As compared with traditional investments, hedge funds may take significant short positions, employ leverage, and engage in very rapid, almost instantaneous, strategy changes with the help of derivatives. Unfortunately, traditional "window-based" regression techniques are limited in their ability to handle these complex investments. To address the limitations of traditional RBSA in dealing with hedge funds, a new methodology, Dynamic Style Analysis (DSA), was specifically developed to improve the returns-based analysis of hedge funds. For our analysis below we use MPI Stylus application utilizing DSA technique.

### Analysis

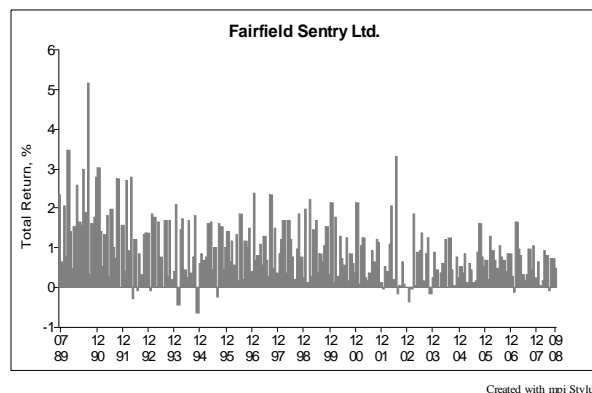
One of the main investment vehicles providing access to Madoff's strategy was the Fairfield Sentry Ltd fund managed by the Fairfield Greenwich Group. The fund ran a so-called "split-strike conversion" strategy. Below

is the description of the fund's strategy derived from the investment memorandum and published in Barron's May 7, 2001 article.

*"Typically, a position will consist of the ownership of 30-35 S&P 100 stocks, most correlated to that index, the sale of out-of-the-money calls on the index and the purchase of out-of-the-money puts on the index. The sale of the calls is designed to increase the rate of return, while allowing upward movement of the stock portfolio to the strike price of the calls. The puts, funded in large part by the sale of the calls, limit the portfolio's downside."*

While there are a number of funds following similar strategy, Madoff's ability to generate stable returns over the long run and in any market environment is perplexing. In this paper we will analyze this fund to better understand the return drivers. For our analysis, we used the returns of the Fairfield Sentry Ltd provided by a third party. The fund's monthly returns are shown in Figure 1.

Figure 1  
**Fairfield Sentry Monthly Returns**



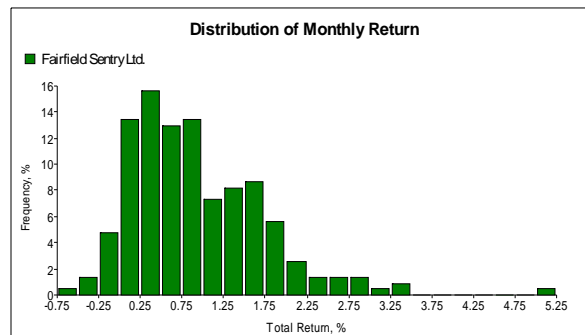
One of the first striking observations one can make is that the fund had very few down months over the 20-year period. The distribution of monthly returns in Figure 2 is positively skewed which is always attractive to investors.

The chart in Figure 3 compares cumulative performance of the fund with the S&P 500 index over almost a 20-year period through September 2008. The fund produced an 11.6% annualized return vs. 9.3% for the S&P 500 index—with much lower volatility. The difference in annual volatility is striking: 2.8% for Sentry and 14% for S&P.

Such a smooth return pattern usually results in a very high Sharpe Ratio and, in fact, this fund and several associated feeder funds have the highest values of long-

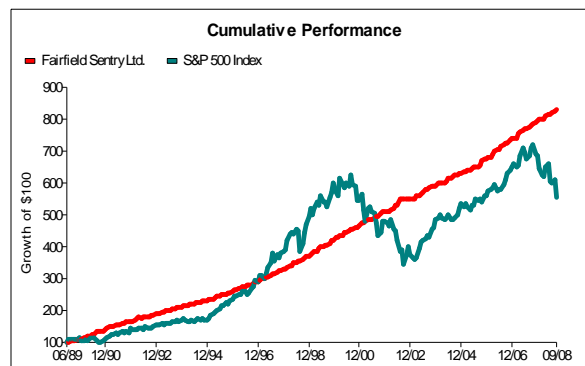
term Sharpe Ratio in the entire 8,000+ hedge fund universe.

Figure 2  
**Distribution of Monthly Return**



Financial ratios such as the Sharpe Ratio, mentioned above as well as the Sortino and Omega ratios and many other similar statistics are routinely used by investors to measure the attractiveness and efficiency of hedge funds. The truth is that these statistics can be misleading as they are academic abstracts and based on certain theoretical assumptions such as normality, etc. As technical indicators, they provide no insight into the internal workings of an active investment strategy, treating it the same way as an individual stock or asset class. Moreover, by providing a false sense of security to investors, such ratios can be very damaging. Thus, for anyone specializing in hedge fund quantitative due diligence, high, almost outlier values for such ratios, are immediately suspect.

Figure 3  
**Cumulative Performance**



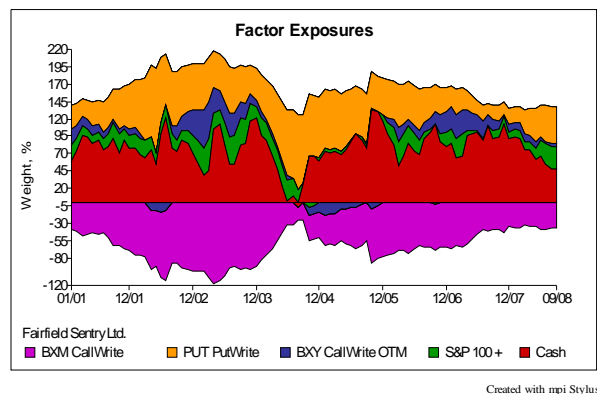
For our regression tests we use various U.S. market indices: S&P 500 GIC sectors, S&P 500, S&P 100 and Russell Equity Style indices. To better detect the fund's option-based strategy we use CBOE S&P 500 BuyWrite indices: "BMX" (at-the-money) and "BXY"

OTM (out-of-the-money), and the CBOE S&P 500 PutWrite “PUT” index.

Once a reasonable set of factors is selected, MPI’s Dynamic Style Analysis (“DSA”) engine goes through various combinations of selected indices to create a dynamic portfolio that provides a reasonably good fit (usually measured by R-squared) and, more importantly, high predictability of results - thus preventing over-fitting the data<sup>2</sup>. Thousands of index combinations were tried over various time intervals but none produced result of any significant credibility. We should note that such a result (or rather lack thereof) is not necessarily an indication of fraud. This could be an indication of missing factors, a strategy that is simply too difficult to model using monthly returns or significant errors in the data. Non-directional strategies, such as stat arbitrage, typically do not produce credible results using RBSA long-term. That being said, it is quite unusual for a fund to not have any credible results over any time interval within a 20- year history.

One of the most interesting results was obtained by using the CBOE options indices<sup>3</sup> as factors. The combination of factors that best explained the Sentry returns was a dynamic portfolio long PUT, S&P 100 and Cash and short BXM and BXY. The weights of this tracking portfolio are shown in Figure 4.

Figure 4  
**Hypothetical Options Strategy**

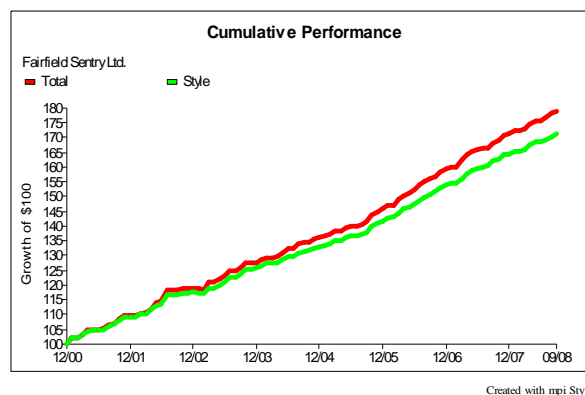


<sup>2</sup> We use MPI’s proprietary Predicted R-squared statistic based on the leave-one-out cross-validation popular in data and image analysis applications. Any data analysis tool should provide a means to prevent overfitting the data. Classical Theil’s Adjusted R-squared is not sufficient and could be misleading.

<sup>3</sup> Although the fund had earlier data, the analysis starts in Jan 2001 as this was the first date of S&P 100 series available to us.

In Figure 5 we compare the fund’s performance (labeled “total”) with the performance of the replication (tracking) portfolio shown above (labeled “Style”). Both performance lines virtually coincide, which signals that the replication portfolio produces nearly identical returns to the fund consistently through the 8-year history.

Figure 5  
**Sentry vs. Hypothetical Strategy**



The result is quite interesting as it is the total opposite of the Barron’s description of the strategy: Madoff is buying puts, selling calls and is also long S&P 100. In our result, the replication portfolio is buying CBOE PutWrite Index, which is equivalent to selling puts and is selling BuyWrite Index which is equivalent to buying calls shorting the index. Note that we haven’t instructed the system to place investments on the long or short side—the data analysis algorithms alone determined that this combination provides the best replication of the fund’s performance.

As mentioned earlier, these results were not deemed credible by our DSA and its predictability measures. This means that small changes in input data could result in significantly different factor exposures. This could signal that we are missing an important factor or not identifying processes that affect fund returns—such as fee deduction, return smoothing, etc. Regardless, the results still raise immediate red flags. For example, given the enormous size of the fund (over \$10B in Sentry and other investments), was it possible to have such a significant volume of option contracts on the long side? Why is the strategy matching the fund’s performance so different from its description by the manager? It is commonly said that RBSA is better at raising questions than it is at answering them, but having the right questions is the most important part of the due diligence process.

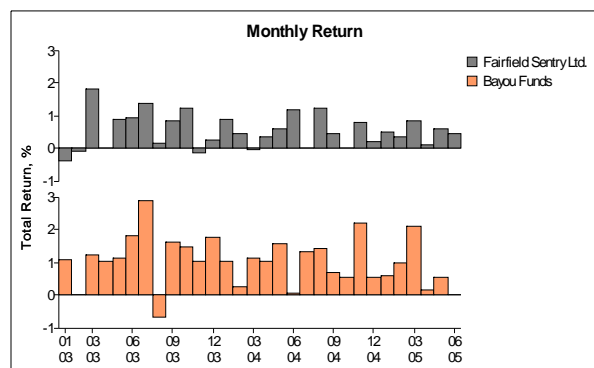
### Adding Missing Factor

Failing to obtain any predictive explanation of the fund's performance using common equity and derivatives indices, we added an unlikely factor—monthly returns of the Bayou fund. Bayou was one of the funds founded by Samuel Israel III, who defrauded investors of hundreds of millions of dollars. As the investment world now knows, Bayou's returns were systematically falsified. Our reason for adding this factor was that characteristics of Madoff's fund and Bayou were strikingly similar—smooth, positive returns in any market environment.

One obvious drawback of our analysis is that data for Bayou is only through June 2005, the last month Bayou reported its performance before filing for bankruptcy. However, the 2.5 years or 30 months used for the analysis is sufficient to draw statistical conclusions about correlation coefficients, while not being too long (as we do not know the exact date that Bayou started inventing its return numbers).

Analysis of both funds' monthly returns in Figure 6 finds that they do have similar patterns in magnitude and the sign of returns: very few negative returns.

Figure 6  
**Bayou and Sentry Monthly Returns**

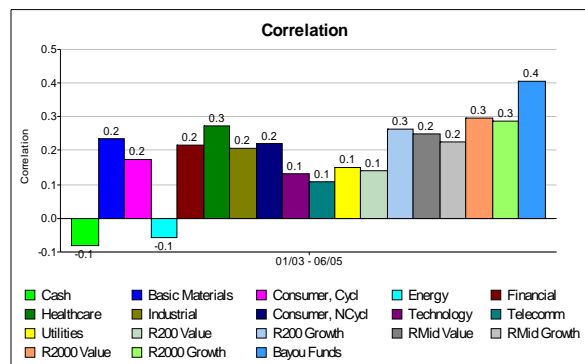


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Further, we measured the correlation of the Fairfield Sentry fund with various sector and style equity indices along with Bayou. Bars in Figure 7 represent the correlation coefficient of Sentry with a respective factor. The rightmost bar represents Sentry's correlation with Bayou, 0.41, which, although is not that high by traditional standards, is still much higher than any other analyzed index. This result is perplexing as the funds' strategy descriptions are strikingly different.

We then compared the returns of 8,000 funds and funds-of-funds from the HFR database to the return streams of both Bayou and Fairfield Sentry Ltd.

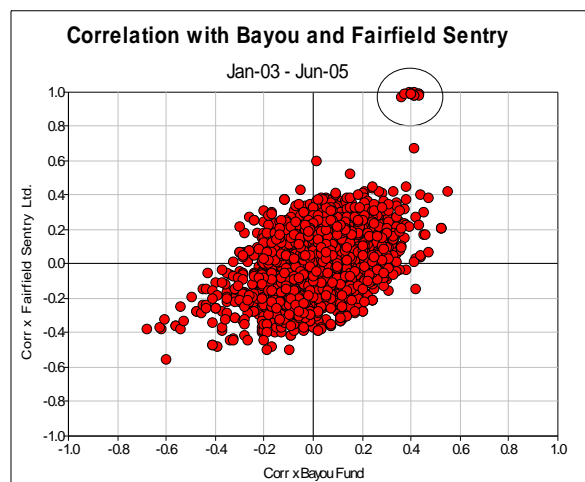
Figure 7  
**Correlation with Fairfield Sentry**



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The results are shown in the scatter chart in Figure 8 below. Each red point represents a hedge fund plotted according to its correlation with the two funds: the X-axis represents correlation with Bayou, the Y-axis correlation with Fairfield Sentry. The cluster circled in the top right corner represents funds that correlate almost perfectly with Fairfield Sentry (correlations close to 1.0). It is plausible to assume that these and other funds having extremely high correlation with Sentry are the feeder funds for Madoff's strategy. Note that these funds also have some of the highest correlation with Bayou's returns.

Figure 8  
**Hedge Fund Correlation with Bayou & Sentry**

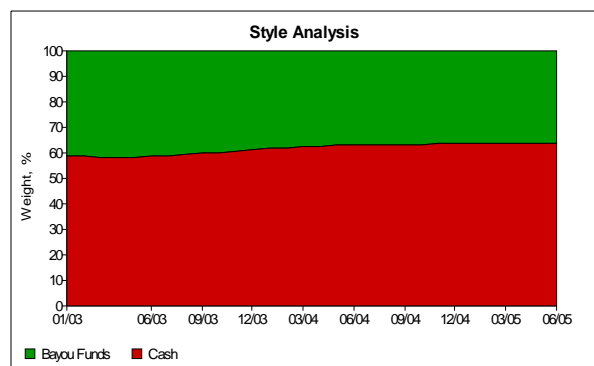


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It shouldn't come as a surprise that a DSA analysis of Madoff's returns with an extended factor set displayed a portfolio made up of cash and the Bayou fund in a very stable combination of 60/40%, respectively, depicted in Figure 9. This also could be described as Fairfield Sentry having beta 0.4 vs. Bayou.

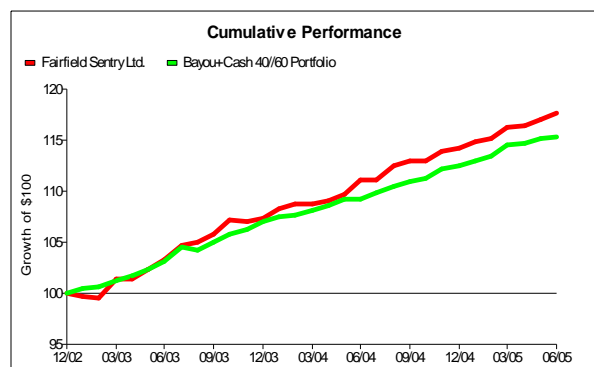
The system rejected any other factor or index as being detrimental to the predictive power of the analysis. Such a stable and simple combination tracked Madoff's fund return fairly closely as is shown in Figure 10.

Figure 9  
**Style Analysis using Bayou as a Factor**



One conclusion from this analysis is that Madoff was 60% less aggressive in marking up numbers than Samuel Israel. On another note, although the Predicted R-squared numbers for this analysis were somewhat low, they were the only positive results obtained using thousands of combinations of indices and factors over various time periods. One immediate question comes to mind: why did such different and unrelated operations produce such strikingly similar returns?

Figure 10  
**Sentry vs. Bayou+Cash Mix**



### Implications for Hedge Fund Indices and Replication Strategies

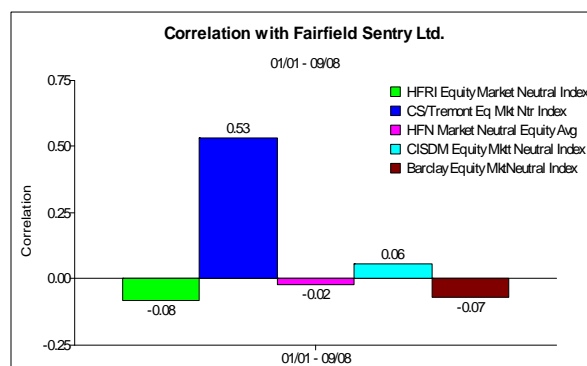
Some of the unlikely casualties of this case are hedge fund indices, which represent important benchmarks of hedge fund performance. A number of hedge fund database vendors<sup>4</sup> compile hedge fund indices by

<sup>4</sup> Credit Suisse/Tremont, Hedge Fund Research (HFR), Hedge Fund.NET, Barclay, CASAM CISDM, Eurekahedge, etc.

strategy. The performance of such indices is usually made public and is updated through the month as more funds report performance. Each category index represents a weighted average of a representative group of funds in the category.

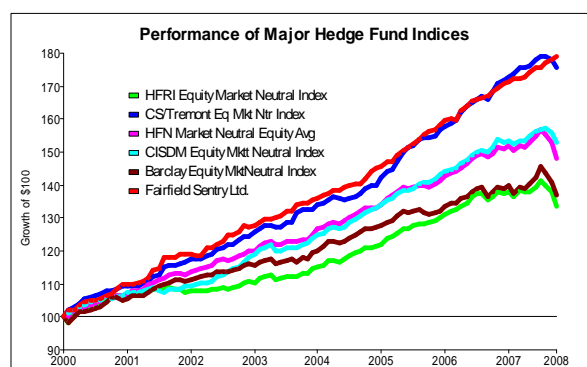
As Madoff's strategy was classified as Market Neutral by the industry, in Figure 11 we compare correlation of the Fairfield Sentry fund with various market neutral indices over the period Jan. 2001–Sept. 2008. The correlation of 0.53 with CS/Tremont Market Neutral index is strikingly high and is, in fact, higher than with any other index.

Figure 11  
**Sentry Correlation with Hedge Fund Indices**



Correlation, representing a measure of linear dependency, is not an immediate indication of whether the fund and the index have similar performance. In fact, even for highly correlated series the actual performance could differ significantly. This was not the case with CS/Tremont, however. In Figure 12 the performance of hedge fund market neutral indices is compared with that of the fund.

Figure 12  
**Performance of Major Hedge Fund Indices**



Performance of the Fairfield Sentry fund represented by a red line consistently overlaps with the CS/Tremont index and mimics it in variability and magnitude. The findings above shouldn't come as a surprise. Certainly, indices differ in coverage and classification. But the major difference affecting index performance lies in its weighting methodology. While most of the hedge fund indices are equal-weighted<sup>5</sup>, CS/Tremont index is asset-weighted, i.e., fund assets under management (AUM) are used as weights when computing index returns. In such an index a fund with significant assets could have a hundred times greater influence on the index performance than a fund with little AUM. This could have a tremendous impact on index performance. Since Madoff's funds accumulated significant assets, an asset-weighted index of which it is a part should be expected to have higher correlation with the fund than an equal-weighted one.

One of the obvious implications for the industry is that these indices will have to be restated. Once such a correction is done, the CS/Tremont index will move closer to the rest of the group.

This case brings both good and bad news to an emerging segment of the industry -- hedge fund replicators. These strategies, available from about a dozen asset managers, aim to track the performance of major hedge fund indices using generic instruments with low fees and high liquidity. Many of these index products have failed to outperform their benchmarks and produced a significant dispersion of results. It is also well known that CS/Tremont indices are notoriously difficult to replicate. And if Madoff related funds' returns contained in them are fraudulent, it is clear why it has been difficult to match their results, so some of these managers should feel relieved.

The bad news is that nobody knows yet how many Madoff and Bayou funds remain out there and are included in hedge fund indices. This reinforces the case that replicating an index has its obvious drawbacks and potential risks. Not only is one replicating a pool of funds where good ideas cancel out bad ones, but also the pool of funds can also contain nonexistent products. As such, it is our suggestion that nothing can replace the thorough due diligence of individual hedge fund returns. Only after such an analysis is performed can one determine if replication is possible and worth doing.

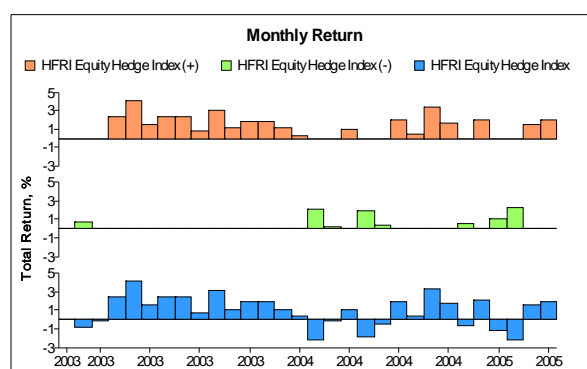
<sup>5</sup> Returns of all funds in the in the index are simply averaged.

## The Anatomy of Fraud

The focus of this section is building a plausible model of behavior for a manager that is systematically marking up performance numbers. One strategy might be "return smoothing", where positive returns are "subsidizing" negative returns. Thus, in a month with a negative return, the fund manager would "adjust" it in a positive direction and in a subsequent month having a positive return, deduct the amount of "adjustment." A simpler strategy would be to simply report returns "as is" when they are positive and report random positive numbers when they are negative. This leads us to the following model.

Let's assume that a manager follows a generic Equity Hedge strategy represented by HFRI Equity Hedge index. From this index series we create two time series: HFRI EH + (plus) and HFRI EH - (minus). The first return series is equal to the original index if the return is positive, and zero if the return is negative. The second, the "minus" series, flips negative returns, i.e., is equal to zero when the index return is positive and to the absolute value of the index return when it is negative. It is easy to see that both time series produce non-negative returns in any market environment and that their difference is equal to the original index. Figure 13 displays all three return series: the truncated positive, flipped negative returns, and original index returns.

Figure 13  
"Improved" HFRI Monthly Returns



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In other words, if  $R$ ,  $R^+$  and  $R^-$  represent index returns, the plus and minus series, respectively, then the following holds:

$$R^+ = \max(R, 0)$$

$$R^- = -\min(R, 0)$$

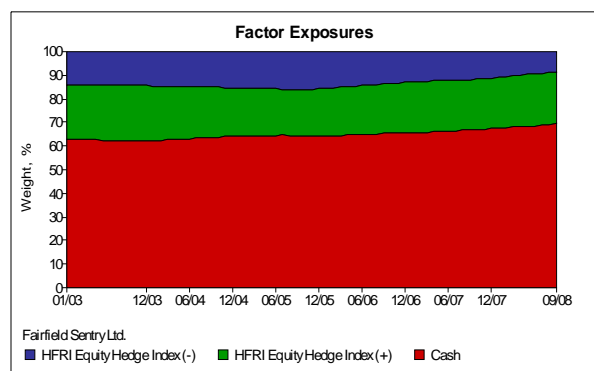
$$R = R^+ - R^-$$

A strategy that produces only positive returns in any market environment  $F$  could be of the following form:

$$F = \beta_1 R^+ + \beta_2 R^- + R^e$$

Where  $\beta_1$  and  $\beta_2$  are opportunistic adjustments that a manager may apply to returns. The higher the numbers, the more aggressive the manager in marking up returns. Our next step is to use this model to see if it could explain the behavior of Madoff in adjusting performance numbers<sup>6</sup>. Thus, we will follow the above model and perform an analysis of Fairfield Sentry returns using  $R^+$ ,  $R^-$  and cash as indices. The result is shown in Figure 14.

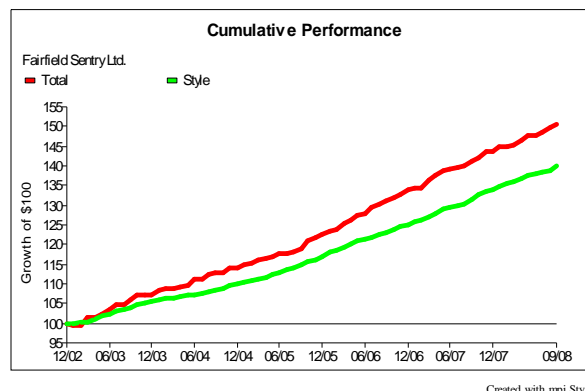
Figure 14  
**Sentry: Hypothetical Strategy Weights**



As is evident from the chart, such a strategy could be explained by taking 30% of returns in up-months, “flipping” negative returns and applying a 20% adjustment in down months. The rest of the portfolio could be invested in a short term instrument, which could also indicate a constant small upward adjustment. This “Style” portfolio tracks the manager fairly well as shown in Figure 15.

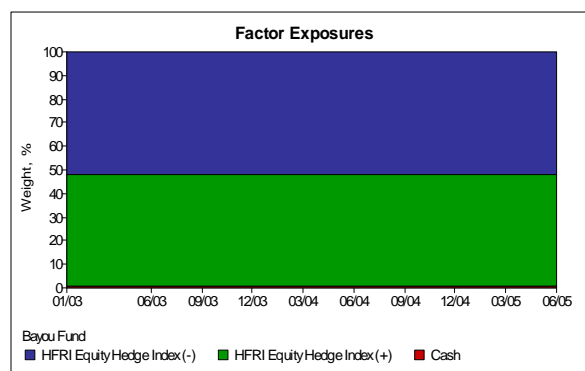
<sup>6</sup> We realize that this model is simplistic. A better model can be built only after obtaining sufficient details about the strategy and the actual holdings. We use this model to detect the trend but we don’t expect to fully capture monthly volatility.

Figure 15  
**Sentry vs. Hypothetical Strategy**



We applied the same approach to Bayou’s returns. The results are shown in Figure 16. Note that Bayou appears more aggressive applying 50% markups in positive and negative months.

Figure 16  
**Bayou: Hypothetical Strategy Weights**



## Conclusion

The use of advanced returns-based style analysis techniques, like DSA, can be instrumental in alerting investors to return patterns and behavior that warrant further investigation. Tools like DSA cannot signal whether a fund is actually engaged in fraudulent activity, but it can definitely raise red flags that should trigger skepticism and a call to action on the part of the hedge fund analyst or investor. Fund performance that is not well explained through returns-based analysis may, of course, be due to missing factors or strategies that are simply too difficult to model with monthly returns. Some of the best hedge funds may not be highly correlated with any standard indices or factors. It is critical that an attempt is made to understand if such unexplained funds are simply difficult to model or if something more sinister is at hand.

Thus, it is our conviction that one must attempt a reconciliation of stated manager strategy and actual fund performance in every case. The case of Madoff and its intriguing similarity and correlation with Bayou indicates that, in some cases, investors can indeed be alerted to *potentially* fraudulent activity in advance. This kind of early warning system is an important tool in the arsenal of a good analyst, but it must be followed up with solid due diligence and investigation on the part

of the analyst to determine if the fund is a keeper or ticking time bomb.

Implications of the Madoff's case for the industry are enormous and may even reach hedge fund index vendors and passive index replicators with both re-examining their rules and due diligence practice.

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### **Media Contacts**

Loretta Mock/Paul Damon

Cognito

+1 646 395 6300

[info@markovprocesses.com](mailto:info@markovprocesses.com)